CITA–94–52

Absolute Ages of Globular Clusters and the Age of the Universe

Brian Chaboyer
Canadian Institute for Theoretical Astrophysics,
60 St. George Street, Toronto, Ontario, Canada M5S 1A7
E-Mail: chaboyer@cita.utoronto.ca

ABSTRACT

The main sequence turnoff luminosity is the best stellar 'clock' which can be used to determine the absolute ages of globular clusters. This is due to the fact that it is generally assumed that the luminosity and lifetimes of main sequence globular cluster stars are independent of the properties of stellar convection and atmospheres, two areas of stellar evolution which are poorly understood. Several possible sources of error in this stellar clock are discussed, and isochrones are constructed using a variety of different physical assumptions. The mean age of the oldest globular clusters are determined from these isochrones and it is found that the uncertainties in the input physics can lead to changes in the derived age of $\pm 15\%$. Surprisingly the largest source of error is the mixing length theory of convection. It is well known that uncertainties in the distance scale and chemical composition of globular cluster stars lead to changes of order $\sim 22\%$ in the determination of absolute ages. Combining the various sources of error, the absolute age of the oldest globular clusters are found to lie in the range 11 — 21 Gyr. This is meant to be a total theoretical range. For the standard inflationary model ($\Omega = 1,\ \Lambda = 0$), a minimum age of the universe of 11 Gyr requires $H_o \lesssim 60\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.

Subject headings: early universe – distance scale – globular clusters: general – stars: interiors – stars: evolution

submitted to *Astrophysical Journal Letters*

1. Introduction

Due to their low metallicity and nearly spherical distribution about the galactic centre, it is evident that the Galactic globular clusters were among the first objects to form in our Galaxy. As such, an accurate determination of the age of the Galactic globular clusters provides a reasonable estimate for the age of the universe. This fact has long been recognized, and determination of globular cluster ages has a rich history (see Demarque, Deliyannis & Sarajedini 1991 for a brief summary). One of the great uncertainties in stellar models is the treatment of convection. Due to this uncertainty, stellar models are not reliable in regions where convection is important. This implies the outer layers of the model for the low mass main sequence stars which make up globular clusters. The cores of these stars (where the nuclear energy generation occurs) are not convective, and so the modeled stellar lifetimes and luminosities are assumed to be unaffected by convection. For this reason, the luminosity of the main sequence turnoff (MSTO) is the best stellar clock with which to determine globular cluster ages (e.g. Sandage 1970). Globular clusters are typically found to have ages of ~ 15 Gyr (Demarque *et al.* 1991; Chaboyer, Sarajedini & Demarque 1992; Bergbusch & VandenBerg 1992; Salaris, Chieffi & Straniero 1993). This has important consequences for cosmology. For example if $\Omega = 1,\ \Lambda = 0$, a minimum age of the universe of 15 Gyr requires $H_o \lesssim 44\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, a value below recent determinations (Freedman *et al.* 1994; Pierce *et al.* 1994; Riess, Press, & Kirshner 1994; Tammann & Sandage 1994). However, before making any judgments regarding the validity of a particular cosmological model based on determinations of H_o and globular cluster ages, one must first understand the associated errors.

Numerous authors have pointed out that the inferred ages of globular clusters determined by the MSTO are quite sensitive to uncertainties in the distance to globular clusters, and the chemical composition of globular cluster stars (Demarque 1980; Rood 1990; VandenBerg 1990; Renzini 1991). However, there has not been a detailed study of errors arising from uncertainties in the physics which are assumed in the stellar models. This *Letter* will examine the possible sources of error in stellar models, and determine how these errors affect the absolute age estimates for globular clusters.

2. Isochrone Construction and Age Determination

For each set of assumed input physics, a series of stellar models with masses ranging from $M = 0.5\ M_\odot$ to $1.0\ M_\odot$ (in $0.05\,M_\odot$ increments) were evolved from the zero-age main sequence to the giant branch using the Yale stellar evolution code (Guenther *et al.* 1992). In order to span the observed range in globular clusters metallicities, the models were evolved with $Z = 6\times10^{-5}$, 2×10^{-4}, 6×10^{-4}, 2×10^{-3}, 4×10^{-3}, and 7×10^{-3}. The following physics was assumed in our standard models: pp reaction rates as tabulated by Bahcall & Pinsonneault (1992); CNO reaction rates as tabulated by Bahcall (1989); high temperature opacities from Iglesias & Rogers (1991); low temperature opacities ($T < 10^4$K) from Kurucz (1991); surface boundary conditions are determined using a grey atmosphere; for temperatures above 10^6 K, a relativistic degenerate, fully ionized equation of state is used; below 10^6 K, the single ionization of ^{1}H , the first ionization of the metals and both ionizations of ^{4}He are taken into account via the Saha equation. The standard models use a solar calibrated mixing length ($\alpha = 1.7$), ^{4}He diffusion is ignored and a ^{4}He abundance of $Y = 0.23$. This is the same set of physics adopted as in the new Yale isochrones (Chaboyer *et al.* 1995).

Observations of globular cluster and halo stars indicate that the α-elements are enhanced over their solar value. This fact was taken into account by modifying the relationship between [Fe/H] and Z (Chaboyer *et al.* 1992; and Salaris *et al.* 1993). For our standard case, the the chosen Z values correspond to [Fe/H] $= -2.8;\ -2.3;\ -1.8;\ -1.3$ all with $[\alpha/\mathrm{Fe}] = +0.40$; [Fe/H] $= -0.9$, $[\alpha/\mathrm{Fe}] = +0.30$; and [Fe/H] $= -0.6$, $[\alpha/\mathrm{Fe}] = +0.25$. These $[\alpha/\mathrm{Fe}]$ values have been chosen to be in agreement with the observations (e.g. Lambert 1989; Brown & Wallerstein 1992; Tomkin *et al.* 1992; King 1993).

Isochrones were constructed by interpolating among the stellar evolutionary tracks to a given age. For each set of input tracks, isochrones were constructed with ages from 9 – 22 Gyr, in 1 Gyr increments. The colour transformation of Green, Demarque & King (1987) was used to convert from theoretical luminosities and colours to observed magnitudes and colours. The effect of changing various of the above assumption was examined by evolving sets of stellar models using different assumptions for the input physics.

In order to compare the isochrones to the observations, one must determine the distance to the globular clusters. This can be done using main sequence fit-

ting, or using the observed absolute magnitude of the RR Lyr stars. Main sequence fitting requires accurate parallaxes of a number of metal-poor stars for an accurate distance determination. Unfortunately, such as sample does not exist at present, and so the absolute magnitude of the RR Lyr stars was adopted as our distance modulus. This allows one to determine the ages using the difference in magnitude between the main sequence turn-off, and the horizontal branch (in the the RR Lyr instability strip). This age determination technique is commonly referred to as ΔV(TO $-$ HB) and has the advantage of being independent of cluster reddening. Unfortunately, RR Lyr stars have convective cores, and so the theoretical calibration of their absolute magnitude is subject to large uncertainties due to the treatment of convection. However, there are several independent observational methods which can be used to determine the absolute magnitude of the RR Lyr stars: (1) Baade-Wesslink and infrared flux methods (Carney, Storm & Jones 1992; Skillen *et al.* 1993) ; (2) statistical parallax observations (Layden *et al.* 1995); (3) the Oosterhoff period-shift effect (Sandage 1993); and (4) determining the apparent magnitude of RR Lyrs in the LMC and using the known distance of the LMC to determine the absolute magnitudes (Walker 1992). These methods have found that the absolute magnitude of the RR Lyr stars is given by:

$$M_v(\mathrm{RRLyr}) = \alpha\,[\mathrm{Fe/H}] + \beta. \qquad (1)$$

Determinations of the slope, α vary from 0.15 to 0.30. Although the slope with metallicity is important in determining the relative ages of the globular clusters, it is not important for deriving the mean absolute age of a number of globular clusters. In this study, $\alpha = 0.22$ was chosen. The absolute age depends sensitively upon the zero-point, β. Galactic determinations of this zero-point agree to within ~ 0.15 mag. However, the LMC RR Lyr calibration is 0.25 mag brighter (Walker 1992). Ages will be derived using both the Walker (1992) and Layden *et al.* (1995) Galactic zero-point which differ by 0.25 mag, and so represent the maximum uncertainty in determining the distance.

The MSTO magnitudes are combined with equation (1) to generate a grid of age (in Gyr, t_9), ΔV(TO $-$ HB) and [Fe/H] which was modeled using an equation of the form

$$\begin{aligned} t_9 &= a_o + a_1\Delta\mathrm{V} + a_2\Delta\mathrm{V}^2 + a_3[\mathrm{Fe/H}] \\ &+ a_4[\mathrm{Fe/H}]^{\,2} + a_5\Delta\mathrm{V}[\mathrm{Fe/H}]\,. \qquad (2) \end{aligned}$$

The rms residuals of the points about the fit were typically 0.15 Gyr. Observationally, it is difficult to determine the MSTO luminosity, as the stars in this region of the colour-magnitude diagram form a nearly vertical strip. Typical errors in determining ΔV(TO$-$HB) are ± 0.14 mag, which translates into an error in the derived age ($\pm 15\%$). To avoid this difficulty, one may determine the mean age for a large number of globular clusters. However, it has become increasingly clear that some globular clusters are significantly younger than the mean. A sample of 24 globular clusters which have nearly the same age and are unequivocoly old has been selected on the basis of their HB morphology (Zinn 1993) and/or the $\Delta(B-V)$ precision age ranking technique (Sarajedini & Demarque 1990; VandenBerg, Bolte & Stetson 1990; Buonanno *et al.* 1994). The observed ΔV(TO $-$ HB) are taken from the compilation of Chaboyer *et al.* (1992). The cluster metallicities are taken from Zinn & West (1984). Equation (2) was used to determine the ages of of these clusters with the error in the age being derived from the observed errors in ΔV(TO$-$HB) and [Fe/H] . The mean age was then formed as a weighted sum of the 24 ages. The error in the mean age was found to be $\pm 3\%$ ($1\,\sigma$).

3. Results

The main results of the *Letter* are summarized in Table 1 where the mean age of the oldest globular clusters is tabulated under a variety of assumptions. Each case will be discussed in turn.

Case A is the standard one, using the assumptions described in the previous section. Note that the uncertainty in the distance modulus of 0.25 mag (Layden vs. Walker age) gives rise to a 4 Gyr change in the derived age. This fact has been noted by many authors (eg. Renzini 1991). It is clear that refining the distance estimate to Galactic globular clusters will play a key role in reducing the uncertainty in the derived ages.

Case B examines the effect of changes in the mixing length, which is used to characterize the transport of energy by convection in the outer layers of the star. Small departures from the solar calibrated mixing length ($\alpha = 1.7$) do not alter the derived age (cases B2 and B3 with $\alpha = 1.5$ and 2.0 change the age by less than 3%). This is to be expected, as it is commonly assumed that the MSTO luminosity is independent of the mixing length. However, if large or small values of the mixing length are considered (cases B1 and B4 with $\alpha = 1.0$ and 3.0), then the derived age of the globular clusters can change by up to 10%. This is due to the fact that changing the mixing length changes the shape of the stellar evolu-

Table 1

Mean Age of the Oldest Globular Clusters[a]

Case	Description	Layden[b] Age (Gyr)	Change	Walker[c] Age (Gyr)	Change
A	**Standard**[d]	$\mathbf{18.2 \pm 0.6}$	—	$\mathbf{14.2 \pm 0.5}$	—
	Changed Physics				
B1	Very Low Mixing length ($\alpha = 1.0$)	16.7	−8.2%	13.4	−5.6%
B2	Low Mixing length ($\alpha = 1.5$)	17.9	−1.6%	14.0	−1.4%
B3	High Mixing length ($\alpha = 2.0$)	18.7	+2.7%	14.4	+1.4%
B4	Very High Mixing length ($\alpha = 3.0$)	20.1	+10.4%	15.4	+8.5%
C	Kurucz color transformation	17.2	−5.5%	13.4	−5.6%
D	Kurucz model atmospheres	18.1	−0.5%	14.1	−0.7%
E	Cox & Stewart low temperature opacities	18.4	+1.1%	14.2	0.0%
F	0.30 H_p adiabatic overshoot at base of scz	18.2	0.0%	14.1	−0.7%
G	LAOL high temperature opacities	18.2	0.0%	14.3	0.7%
H1	$3\,\sigma$ decrease in nuclear reaction rates	19.1	+4.9%	14.8	+4.2%
H2	$2\,\sigma$ decrease in nuclear reaction rates	18.6	+2.2%	14.5	+2.1%
H3	$2\,\sigma$ increase in nuclear reaction rates	17.9	−1.6%	13.9	−2.1%
H4	$3\,\sigma$ increase in nuclear reaction rates	17.8	−2.2%	13.8	−2.8%
J	Debyre-Hückel EOS	17.0	−6.6%	13.3	−6.3%
K	^{4}He Diffusion	16.9	−7.1%	13.1	−7.7%
	Changed Abundances				
L1	low ^{4}He abundance ($Y = 0.20$)	19.1	+4.9%	14.9	+4.9%
L2	high ^{4}He abundance ($Y = 0.26$)	17.5	−3.8%	13.5	−4.9%
M1	GC [Fe/H] decreased by 0.10 dex	18.4	+1.1%	14.3	+0.7%
M2	GC [Fe/H] increased by 0.10 dex	18.1	−0.5%	14.0	−1.4%
N1	$[\alpha/\mathrm{Fe}] = +0.6$	17.1	−6.0%	13.2	−7.0%
N2	$[\alpha/\mathrm{Fe}] = +0.2$	19.4	+6.6%	15.1	+6.3%

[a]Based on their HB morphology (Zinn 1993) and/or the $\Delta(B-V)$ age precision age ranking technique. There are 24 clusters in this group with measured ΔV(TO − HB) values. The clusters span a range in metallicity of $-0.89 \leq$ [Fe/H] ≤ -2.41.

[b]Ages derived using the Layden *et al.* (1995) RR Lyr distance scale

[c]Ages derived using the Walker (1992) RR Lyr distance scale

[d]The input physics and composition for this model is discussed in §2.

tionary models, and hence the isochrones. Thus, the luminosity of the bluest point on the isochrone can be altered, even though the age-luminosity relationship is relatively unaffected by changes in the mixing length. We conclude that uncertainties in how to treat convection in stellar models leads to a maximum uncertainty of 10% in globular cluster ages derived using the MSTO luminosity. This is one of the largest sources of error the age estimates. The $\alpha = 1.0$ and $\alpha = 3.0$ isochrones do not match observed globular cluster colour-magnitude diagrams. Thus, one would be tempted to state that these isochrones (and the associated large uncertainty in globular cluster ages) are ruled out by the observations. However, there are other uncertainties that can strongly affect the colour of the models (surface boundary conditions, opacities, colour transformation). In addition, there is no compelling reason that the mixing length should be the same for the Sun and globular cluster stars, or even have the same value on the main sequence and the giant branch. For these reasons, one should not rule out the $\alpha = 1.0$ and $\alpha = 3.0$.

Case C considers the effect of using the Kurucz (1992) colour calibration to convert from the modeled luminosities and temperatures to observed colours and temperatures. Use of the Kurucz (1992) colour calibration reduces the derived age by 5%. Determining colour calibrations is extremely difficult, and comparing colour calibrations which have been independently derived gives a good estimate for the error involved in the process.

Case D uses the Kurucz (1992) model atmospheres for the surface boundary conditions (the standard case using a grey atmosphere). The change is the derived age is very small ($< 0.6\%$), and so uncertainties in surface boundary conditions used in stellar models have little effect on our estimate for the absolute age of the globular clusters.

Case E uses the Cox & Stewart (1970) opacities for $T < 10^4$ K as opposed to the Kurucz (1991) opacities used in the standard case. This change has very little effect on the derived ages, as is to be expected from the fact that only the surface layers of the star are affected by changes in the low temperature opacities.

Case F includes the effect of a rather large (0.30 pressure scale heights) adiabatic overshoot layer at the base of the surface convection zone. This has very little effect on the derived age. Together, cases D, E, and F demonstrate that the physical conditions in the core of the star are unaffected by changes in the outer layers of the star.

Case G uses the LAOL opacities of Huebner *et al.* (1977) for the interior of the model ($T > 10^4$K). The derived age is very similar to the standard model, which uses the opacities of Iglesias & Rogers (1991). Thus, uncertainties in the calculation of opacities have little effect on globular cluster ages derived using the MSTO luminosity.

Case H investigates the effects of changing the nuclear reaction rates. Even changing all of the nuclear reaction rates by $3\,\sigma$ from their tabulate values changes the derived age by less than 5%. Changing the reaction rates by $2\,\sigma$ is perhaps a more more realistic assessment of the possible error, and indicates that the possible errors in the nuclear reaction rates have a negligible effect ($\sim 2\%$) on the derived age. For these low mass, low metallicity stars, the key nuclear energy generating reaction is the pp chain, and the small change in the derived age due to changing the nuclear reactions is a reflection of the fact that the pp cross section is well understood on theoretical grounds (Bahcall 1989).

Case J includes the effects of the Debyre-Hückel correction to the equation of state (which takes into account the Coulomb forces, see Guenther *et al.* 1992 for a description of its implementation). It leads to a decrease in the derived age of 6.5%. However, the Debyre-Hückel correction is not the only non-ideal gas effect which should be included in a realistic equation of state. Rogers (1981; 1986) discusses various corrections to the equation of state in stellar plasmas, and finds that other effects may be as important (and have the opposite sign) as the Debyre-Hückel. For this reason, it is not clear that including the Debyre-Hückel correction results in an equation of state which better reflects the true equation of state. We conclude that uncertainties in the equation of state can lead to changes in the derived age of the oldest globular clusters of $\sim \pm 4\%$.

Case K includes the effect of the diffusion of ^{4}He relative to ^{1}H. For this study, the diffusion coefficients of Michaud & Proffitt (1993) were used and the age of the oldest globular clusters is reduced by 7%. The effects of diffusion on the ages of globular clusters have been studied by numerous authors (e.g. Noerdlinger & Arigo 1980; Proffitt & VandenBerg 1991; Chaboyer *et al.* 1992), and our results are compatible with the later studies. However, as pointed out by Chaboyer & Demarque (1994), stellar models which only include diffusion are unable to explain the observed plateau in Li abundances which occurs in halo stars. For this reason, it is unclear if diffusion actually occurs in stars, but it does remain a possible source of error in the stellar models.

Cases L – N demonstrate the effects that the uncertainty in the true chemical composition of globular cluster stars have on the derived age estimate. This effect has been studied by others (e.g. Demarque 1980; VandenBerg 1990; Renzini 1991), and so is only briefly discussed here. Changing the ^{4}He abundance or an overall shift in the [Fe/H] scale for globular clusters has little effect on the derived age. However, the uncertainty in the true abundance of the α-capture elements (and oxygen in particular) leads to changes in the derived age of the oldest globular cluster of $\pm 7\%$. Deriving oxygen abundances from observations is subject to a number of systematic errors, and a range of [O/Fe] $= +0.2$ to 0.6 dex which was used in the calculations is indicative of the scatter found between different observers (see King 1993).

Although not tabulated in Table 1, a number of cases have been constructed which combine several of the above effects and it was found that the percent change in the derived age is well approximated by adding up the percentage change for the individual cases considered.

4. Discussion

The mean age for the oldest globular globular clusters derived using the MSTO luminosity is tabulated in Table 1 for a variety of different physical assumptions. Aside from the 22% change in the derived age which arises from determining the true distance to the cluster (the 'Layden' and 'Walker' ages in Table 1), the largest single change in the age determination ($\pm 10\%$) is due to our poor understanding on how to model convection in stellar evolution models. This is exemplified by case B which considered different values of the mixing length. This is a somewhat surprising result, as it is commonly assumed that ages derived using the MSTO luminosity are insensitive to uncertainties in our knowledge of convection. Other variables which can lead to large changes in the derived age include the colour transformation (case C, 5%), the equation of state (case J, 7%) and whether or not to include helium diffusion (case K, 7%). This is comparable to the error in the age due to the uncertain α-element composition (case N, $\pm 7\%$). It is impossible to quantify a $1\,\sigma$ error in the derived age of the globular clusters, as many of the sources of error are systematic. However, one can conclude that even if the distance and compositions of the globular clusters were known exactly, the total uncertainty in the age estimate would be $\pm 15\%$.

The effects of mass loss or rapidly rotating cores has not been considered in this work. Shi (1994) found that the age reduction due to mass loss was constrained by the observations to be less than ~ 1 Gyr. The effect of rapidly rotating cores on stellar models and isochrones has been studied by Deliyannis, Demarque & Pinsonneault (1989) and Chaboyer & Demarque (1994), who found that rotation has virtually no effect on the derived ages. In light of these results, and of Table 1, one may conclude that the true age of globular clusters lies in the range 11 – 21 Gyr. In the context of cosmology, it is the lower limit which is of most interest. To reach this lower limit a number of non-standard assumptions must be made: ^{4}He diffusion occurs in stars; the Debyre-Hückel correction should be included in the equation of state; mass loss is occurring in main sequence globular cluster stars and the galactic calibration of the RR Lyr absolute magnitudes must be in error by 0.25 magnitudes. Although it is possible that a few of these effects are true, it would seem unlikely that they all occur. Based on this, a more realistic assessment of the globular cluster ages would be 13 – 17 Gyr.

A lower limit of 11 or 13 Gyr to the age of the universe imposes strong constraints on the standard inflation model ($\Omega = 1$, $\Lambda = 0$). A minimum age of 11 Gyr requires $H_o \lesssim 52$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$, while 13 Gyr requires $H_o \lesssim 60$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Recent measurements of Cepheids in Virgo by CFHT (Pierce *et al.* 1994) and HST (Freedman *et al.* 1994) find $H_o = 87 \pm 7$ and 80 ± 17 respectively. If the central value remains the same, as the error in the determination of H_o is reduced, an $\Omega = 1$, $\Lambda = 0$ universe would be ruled out. A low density $\Omega = 0.1$, $\Lambda = 0$ universe is compatible with an age of 11 Gyr and $H_o \simeq 80$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

I would like to thank A. Layden for providing me with his RR Lyr calibration and R. Zinn who shared with me his list of old halo clusters, both in advance of publication. I am grateful to P. Demarque and R. Malaney for their comments on an early draft of this paper.

REFERENCES

Bahcall, J.N. 1989, Neutrino Astrophysics (Cambridge: Cambridge Univ. Press)

Bahcall, J.N., & Pinsonneault, M.H. 1992, Reviews of Modern Physics, 64, 885

Bergbusch, P.A. & VandenBerg, D.A. 1992, ApJS, 81, 163

Buonanno, R., Corsi, C.E., Fusi Pecci, F., Fahlman, G. & Richer, H.B. 1994, ApJ, 430, L121

Brown, J.A. & Wallerstein, G. 1992, AJ, 104, 1818

Carney, B.W., Storm, J. & Jones, R.V. 1992, ApJ, 386, 663

Chaboyer, B. & Demarque, P. 1994, ApJ, 433, 510

Chaboyer, B., Demarque, P., Guenther, D.B., Pinsonneault, M.H. & Pinsonneault, L.L. 1995, to appear in The Formation of the Milky Way, eds. E.J. Alfaro & G. Tenorio-Tagle (Cambridge: Cambridge University Press)

Chaboyer, B., Sarajedini, A. & Demarque, P. 1992, ApJ, 394, 515

Cox, A.N. & Stewart, J.N. 1970, ApJS, 19, 261

Deliyannis, C.P., Demarque, P. & Pinsonneault, M.H. 1989, ApJ, 347, L73

Demarque, P. 1980, in Star Clusters, IAU Symp. 85, ed. J.E. Hesser (Dordrecht: Reidel), 281

Demarque, P., Deliyannis, C.P. & Sarajedini, A. 1991, in Observational Tests of Cosmological Inflation, eds. T. Shanks, *et al.* (Dordrecht: Kluwer), 111

Freedman, W.L. *et al.* 1994, Nature, 371, 757

Green, E.M., Demarque, P. & King, C.R. 1987, The Revised Yale Isochrones & Luminosity Functions (New Haven: Yale Univ. Obs.)

Guenther, D.B., Demarque, P., Kim, Y.-C., & Pinsonneault, M.H. 1992, 387, 372

Huebner, W.F., Merts, A.L., Magee, N.H., & Argo, M.F. 1977, Los Alamos Opacity Library, Los Alamos Scientific Laboratory Report, No. LA–6760–M

Iglesias, C.A. & Rogers, F.J. 1991, ApJ, 371, 408

King, J.R. 1993, AJ, 106, 1206

Kurucz, R.L. 1991, in Stellar Atmospheres: Beyond Classical Models, ed. L. Crivellari, I. Hubeny, D.G. Hummer, (Dordrecht: Kluwer), 440

Kurucz, R.L. 1992, in IAU Symp. 149, The Stellar Populations of Galaxies, ed. B. Barbuy, A. Renzini, (Dordrecht: Kluwer), 225

Lambert, D.L. 1989, in Cosmic Abundances of Matter, ed. C.J. Waddington (New York: AIP), 168

Layden, A.C., Hawley, S.L., Hanson, R.B. & Klemola, A.R. 1995, in preparation

Michaud, G. & Proffitt, C.R. 1993, in Inside the Stars, IAU Col. 137, ed. A. Baglin & W.W. Weiss (San Fransico: PASP), 246

Noerdlinger, P.D., & Arigo, R.J. 1980, ApJ, 237, L15

Pierce, M.J., Welch, D.L., McClure, R.D., van den Bergh, S., Racine, R. & Stetson, P.B. 1994, Nature, 371, 385

Proffitt, C.R., & VandenBerg, D.A. 1991, ApJS, 77, 473

Renzini, A. 1991, in Observational Tests of Cosmological Inflation, eds. T. Shanks, *et al.* (Dordrecht: Kluwer), 131

Riess, A.G., Press, W.H. & Kirshner, R.P 1994, submitted to ApJ Letters

Rogers, F.J. 1981, Phys Rev, A24, 1531

Rogers, F.J. 1986, ApJ, 310, 723

Rood, R.T. 1990, in Astrophysical Ages and Dating Methods, eds. E. Vangioni-Flan *et al.* (Gif sur Yvette: Ed. Frontières), 313

Salaris, M. Chieffi,A. & Straniero, O. 1993, ApJ, 414, 580

Sarajedini, A. & Demarque, P. 1990, ApJ, 365, 219

Sandage, A. 1970, ApJ, 162, 841

Sandage, A. 1993, AJ, 106, 719

Shi, X. 1994, submitted to ApJ (FERMILAB-Pub-94/348-A)

Skillen, I., Fernley, J.A., Stobie, R.S. & Jameson, R.F. 1993, MNRAS, 265, 301

Tammann, G.A. & Sandage, A. 1994, submitted to ApJ

Tomkin, J., Lemke, M., Lambert, D.L. & Sneden, C. 1992, AJ, 104, 1568

VandenBerg, D.A. 1990, in Astrophysical Ages and Dating Methods, eds. E. Vangioni-Flan *et al.* (Gif sur Yvette: Ed. Frontières), 241

VandenBerg, D.A., Bolte, M. & Stetson, P.B. 1990, AJ, 100, 445

Walker, A.R. 1992, ApJ, 390, L81

Zinn, R. 1993, in The Globular Cluster-Galaxy Connection, eds. G.H. Smith & J.P. Brodie (San Francisco: ASP), 38

Zinn, R. & West, M. 1984, ApJS, 55, 45

This 2-column preprint was prepared with the AAS LaTeX macros v3.0.